

Introduction to Three Dimensional Geometry

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Pankaj and his father were walking in a large park. They saw a kite flying in the sky. The position of Kite, Pankaj and Pankaj's father are at $(20, 30, 10)$, $(4, 3, 7)$ and $(5, 3, 7)$ respectively.



(A) The distance between Pankaj and Kite is:

- (a) 41.32 units
- (b) 31.52 units
- (c) 38.32 units
- (d) 40.39 units

(B) The distance between Pankaj's father and kite is:

- (a) 31.30 units
- (b) 38.43 units
- (c) 31.03 units
- (d) 29.00 units

(C) The co-ordinates of Pankaj lie in:

- (a) IV quadrant
- (b) III quadrant
- (c) II quadrant
- (d) I quadrant

(D) If co-ordinate of Kite, Pankaj and Pankaj's father form a triangle, then the centroid is:

- (a) $(9.67, 12, 8)$



(b) (9.6, 8, 12)

(c) (12, 8, 10)

(d) (7, 9, 7.2)

(E) The co-ordinates of points in the XY-plane are of the form:

(a) (0, 0, z)

(b) (x, y, 0)

(c) (x, 0, y)

(d) (0, x, y)

Ans. (A) (b) 31.52 units

Explanation: Required distance

$$\begin{aligned} &= \sqrt{(20-4)^2 + (30-3)^2 + (10-7)^2} \\ &= \sqrt{16^2 + 27^2 + 3^2} \\ &= \sqrt{256 + 729 + 9} \\ &= \sqrt{994} \\ &= 31.52 \text{ units} \end{aligned}$$

(B) (c) 31.03 units

Explanation: Required distance

$$\begin{aligned} &= \sqrt{(20-5)^2 + (30-3)^2 + (10-7)^2} \\ &= \sqrt{15^2 + 27^2 + 3^2} \\ &= \sqrt{225 + 729 + 9} \\ &= \sqrt{963} \\ &= 31.03 \text{ units} \end{aligned}$$

(C) (d) I quadrant

Explanation: Because in (4, 3, 7); all are positive.

Thus, the coordinate lies in the I quadrant.

(D) (a) (9.67, 12, 8)

Explanation: Centroid

$$= \left(\frac{20+4+5}{3}, \frac{30+3+3}{3}, \frac{10+7+7}{3} \right)$$

= (9.67, 12, 8)

(E) (b) (x, y, 0)



Explanation: For XY-plane, $z = 0$

$\Rightarrow$ The co-ordinates are of the form $(x, y, 0)$.

2. Vikas and his friends went camping for 2 nights and 3 days. There they set up a tent which is triangular in shape. The vertices of the tent are $A(4, 5, 9)$, $B(3, 2, 5)$, $C(5, 2, 5)$, $D(-3, 2, -5)$ and $E(-4, 5, -9)$ respectively.



The vertex A is tied up by the rope at the ends F and G and the vertex E is tied up at the ends I and H.

(A) If M denotes the position of their bags inside the tent and it is just in middle of the vertices B and D, then find the coordinates of M and the length AE.

(B) If the length of the rope by which E is tied up with H is $5\sqrt{2}$ units, then find the equation denotes the set of point of H and the length BC.

(C) Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$.

Ans. (A) As, M is the middle point of $B(3, 2, 5)$ and $D(-3, 2, -5)$

$\therefore$ The coordinates of M are

$$\left(\frac{3-3}{2}, \frac{2+2}{2}, \frac{5-5}{2} \right) = (0, 2, 0)$$

The length AE is

$$= \sqrt{(-4-4)^2 + (5-5)^2 + (-9-9)^2}$$

$$= \sqrt{64+0+324}$$

$$= \sqrt{388}$$

$$= 2\sqrt{97} \text{ units}$$

(B) As, the distance of $H(x, y, z)$ from $E(-4, 5, -9)$ is $5\sqrt{2}$ units.

$\therefore EH = 5\sqrt{2}$

$$\Rightarrow \sqrt{(x+4)^2 + (y-5)^2 + (z+9)^2} = 5\sqrt{2}$$

On squaring both sides, we get

$$(x+4)^2 + (y-5)^2 + (z+9)^2 = 25 \times 2$$

$$x^2 + y^2 + z^2 + 8x - 10y + 18z + 122 = 50$$

$$\Rightarrow x^2 + y^2 + z^2 + 8x - 10y + 18z + 72 = 0$$

The length BC is,

$$BC = \sqrt{(5-3)^2 + (2-2)^2 + (5-5)^2}$$

$$= \sqrt{4+0+0}$$

$$= 4 \text{ units}$$

(C) Assume that $P(x, y, z)$ be the point that is equidistant from two points $A(1, 2, 3)$ and $B(3, 2, -1)$.

Thus, we can say that, $PA = PB$

Take square on both the sides, we get

$$PA^2 = PB^2$$

It means that,

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

Now, simplify the above equation, we get

$$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$$

$$\Rightarrow -2x - 6z + 6x - 2z = 0$$

$$\Rightarrow 4x - 8z = 0$$

$$\Rightarrow x - 2z = 0$$